

Emmanuel Junior Academy (DSAT)

MATHEMATICAL CALCULATION AND REPRESENTATION GUIDE

Emmanuel Junior Academy Mathematical Calculation and Representation Guide

Our Mathematical Calculation and Representation Guide is adapted from the White Rose Maths calculation policy, which is a high-quality resource used in our school to support effective teaching and learning of the ambitious Primary Maths National Curriculum. Our guide focuses on ensuring that pupils learn the most appropriate and efficient methods of calculation at the right time.

A common approach to manipulatives, representations and methods of calculation ensures that there is clear progression for pupils through school. Methods and representations build on from the year before to ensure that pupils make incremental, small steps in their maths learning, leading to secure understanding.

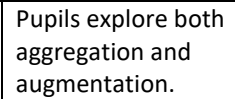
Our Calculation and Representation Guide also supports teachers to make adaptations to their teaching when there are pupils working below Age Related Expectations. A clear and shared understanding representations and methods from lower down school (including KS1 where required), ensures that all teachers can consistently provide high-quality teaching and learning for all pupils.

Throughout KS2, pupils are challenged to calculate using numbers of increasing size and complexity (including decimals). The representations and calculation methods in this guide support pupils to develop an understanding of and use the following methods effectively by the end of the year.

By the end of the year, pupils will use these methods effectively to calculate...				
	Addition	Subtraction	Multiplication	Division
Year 3	Formal column addition	Formal column subtraction	Informal written methods of multiplication Short multiplication	Division as sharing with an exchange, using informal written methods.
Year 4	Formal column addition	Formal column subtraction	Short multiplication	Informal written methods of division, including remainders.
Year 5	Formal column addition	Formal column subtraction	Short multiplication Long multiplication	Short division with remainders.
Year 6	Formal column addition	Formal column subtraction	Long multiplication	Formal method of short division. Formal method of long division. Remainders as fractions or decimals.

Please see the glossary of Key Terms and Examples at the end of this document for clarification and exemplifications of formal methods and layout expectations.

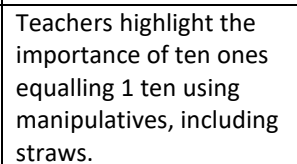
Add 1-digit numbers within 10



Aggregation: part-whole model, discrete and continuous bar model, number shapes, tens frame

Augmentation:
combination bar model, ten
frame, bead string, number
track

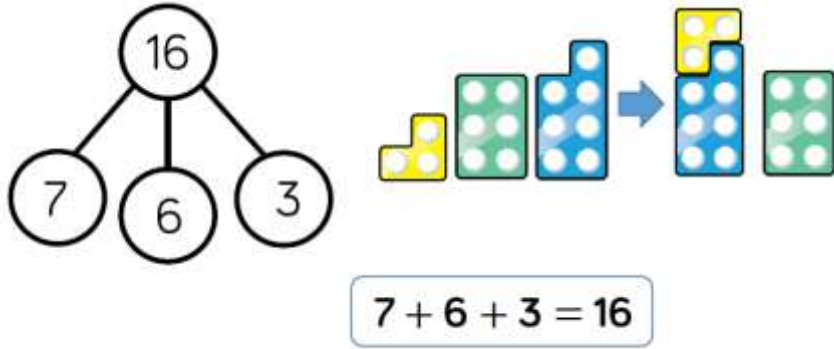
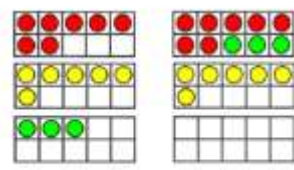
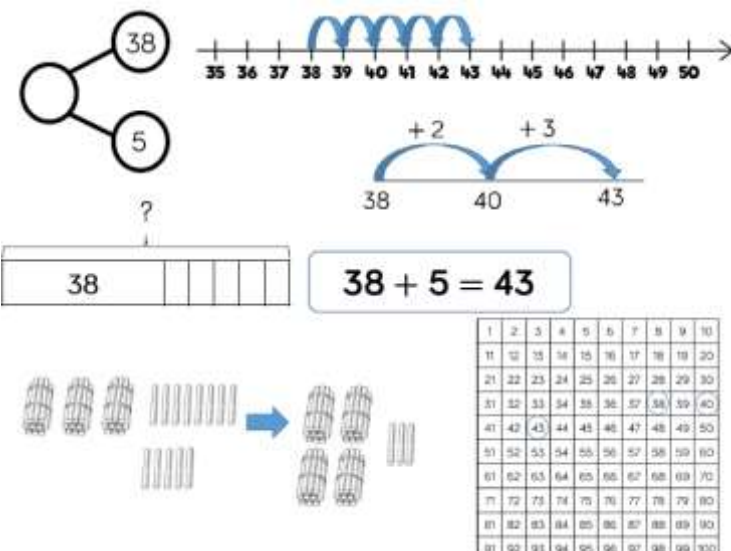
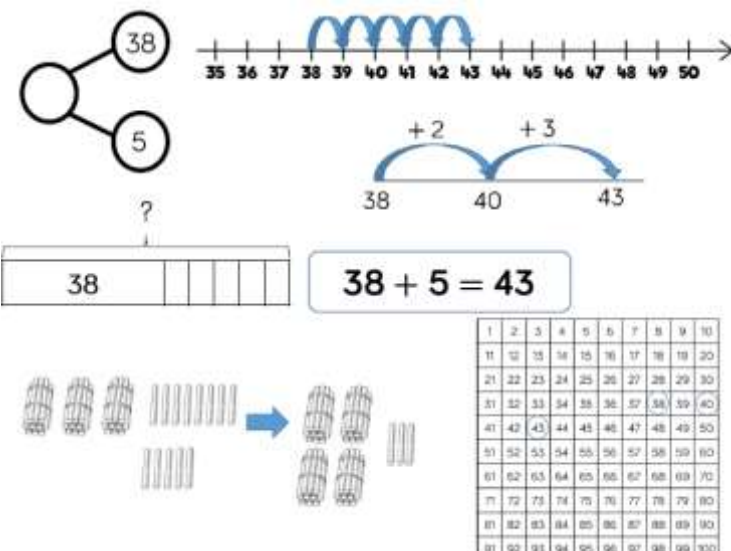
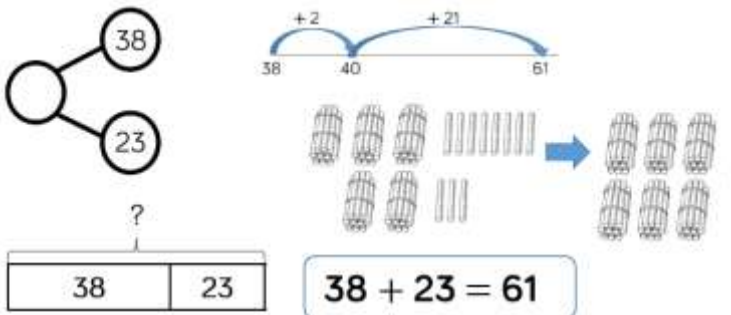
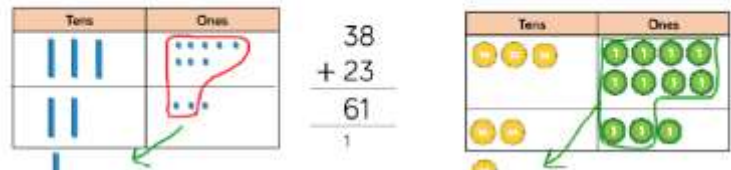
Add 1 and 2-digit numbers to 20



When adding 1-digit numbers that cross 10, pupils can regroup manipulatives.

Pupils use concrete resources alongside number lines to understand how to partition their jumps.

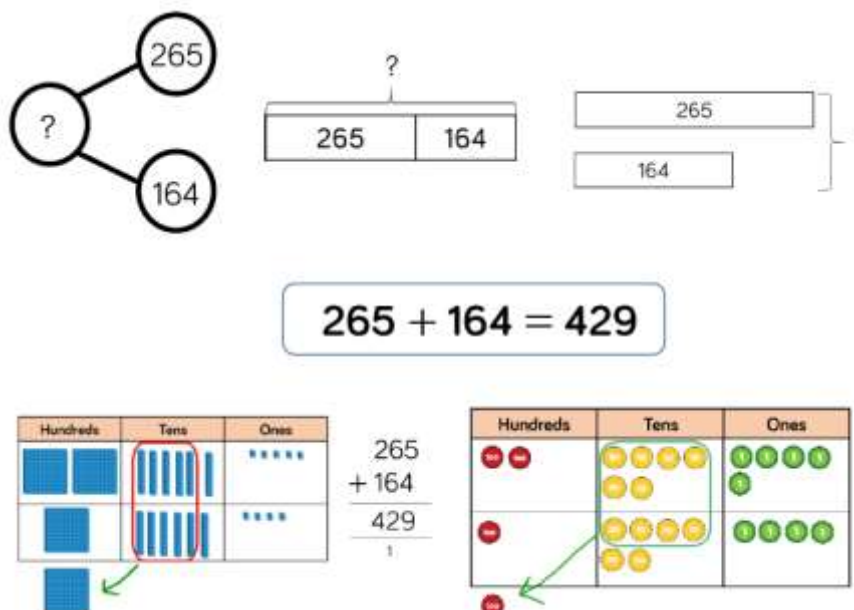
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Year 2	Add 3 1-digit numbers  $7 + 6 + 3 = 16$  $7 + 6 + 3 = 16$  $38 + 5 = 43$	Pupils use known facts, such as number bonds to 10 and doubles to support them to add quickly. Pupils are exposed to a range of manipulatives that highlight bonds to 10 are effective when adding 1-digit numbers.
Year 2/3	Add 1-digit and 2-digit numbers to 100  $38 + 5 = 43$	Pupils count on from the larger number when adding 2+1 digit numbers. They also apply their knowledge of number bonds to add more efficiently e.g. $8+5 = 13$ so $38 + 5 = 43$ Hundred squares and straws can support children to find the number bond to 10.
Year 2/3	Add 2-digit and 2-digit numbers to 100  $38 + 23 = 61$  $38 + 23 = 61$	Children are encouraged to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient. Children also use a blank number line to count on to find the total. Pupils jump into multiples of 10 as they become more efficient. By the end of Year 2 pupils will use formal column addition including regrouping.

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Year 3

Add numbers with up to 3-digits:



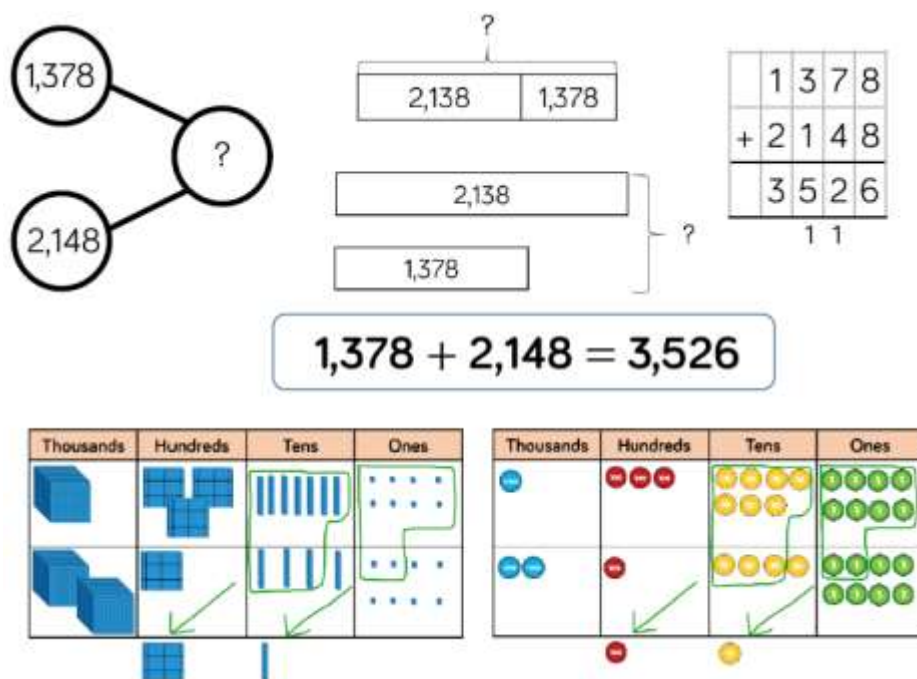
Pupils use Base 10 and place value when adding numbers with up to 3-digits.

Pupils write the formal written method of column addition alongside any representations/concrete resources so they can make links between the two.

In column addition, pupils record regrouped digits underneath the equals sign.

Year 4

Add numbers with up to 4-digits:



Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 4-digits.

Pupils should write the calculation alongside any concrete resources to see the links.

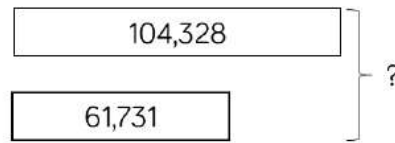
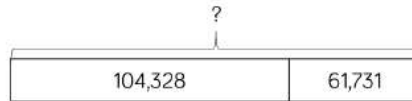
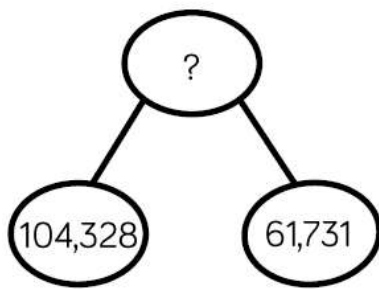
Plain counters on a place value grid also support learning.

In column addition, pupils record regrouped digits underneath the equals sign.

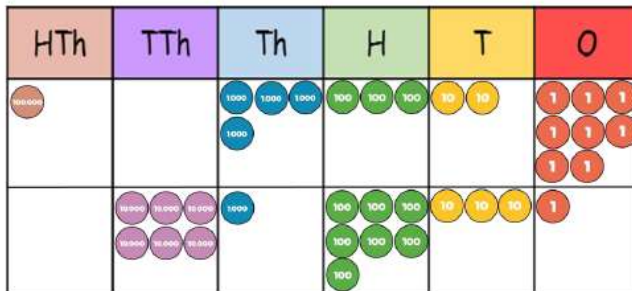
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Year 5/6

Adding numbers with more than 4-digits



$$104,328 + 61,731 = 166,059$$



1	0	4	3	2	8
+	6	1	7	3	1
1	6	6	0	5	9

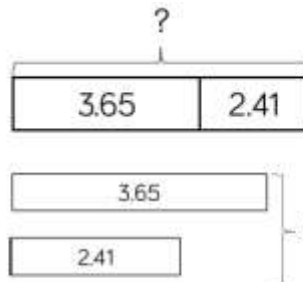
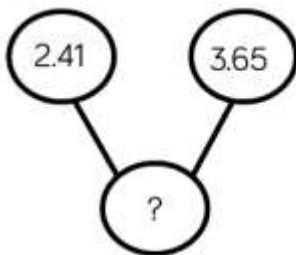
Pupils use the column method of addition to add large numbers efficiently.

In column addition, pupils consistently record regrouped digits underneath the equals sign.

In Year 6, pupils explain why column addition may not be suitable for every calculation, and can make choices to use efficient mental methods.

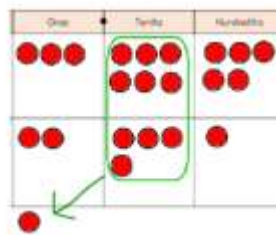
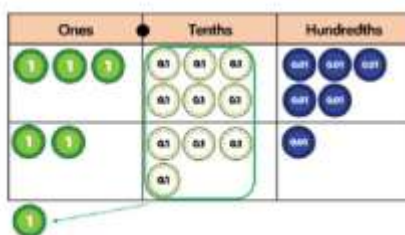
Year 5/6

Add with up to 3 decimal places



$$\begin{array}{r} 3.65 \\ + 2.41 \\ \hline 6.06 \\ 1 \end{array}$$

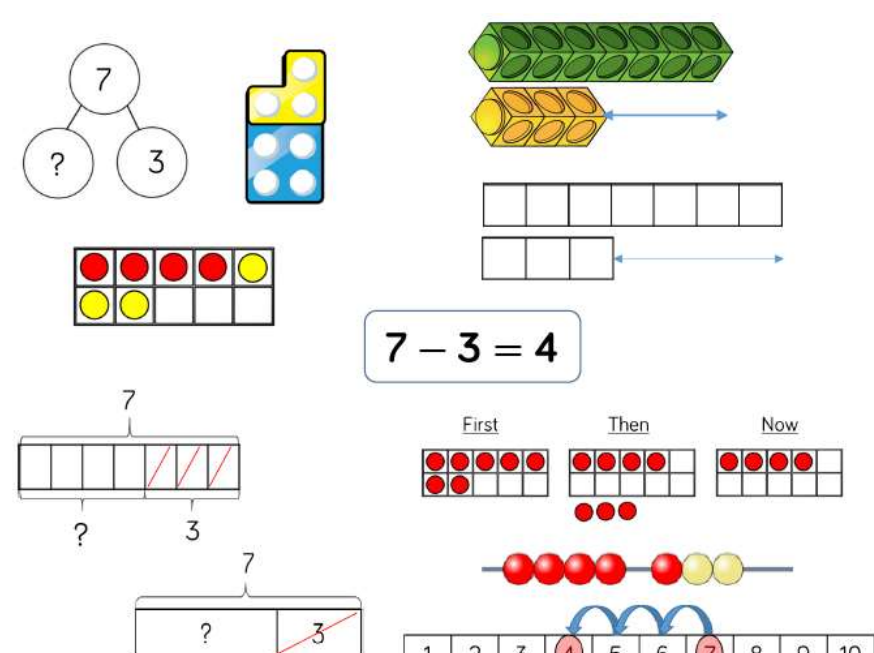
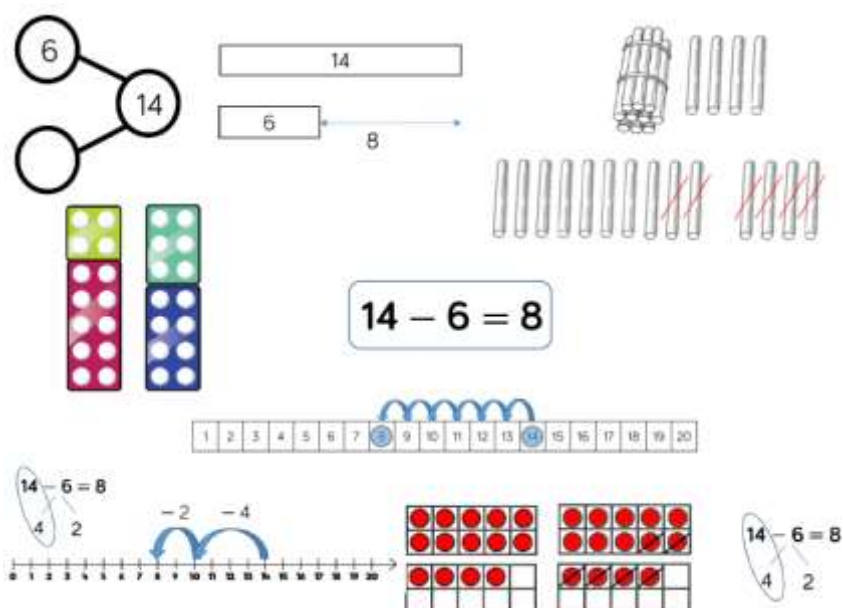
$$3.65 + 2.41 = 6.06$$



Children have experience of adding decimals with a variety of decimal places. This includes putting this into context when adding money and other measures.

In column addition, pupils consistently record regrouped digits underneath the equals sign.

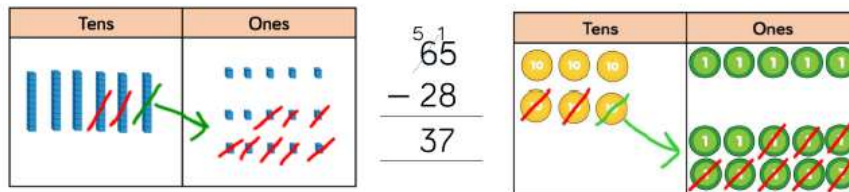
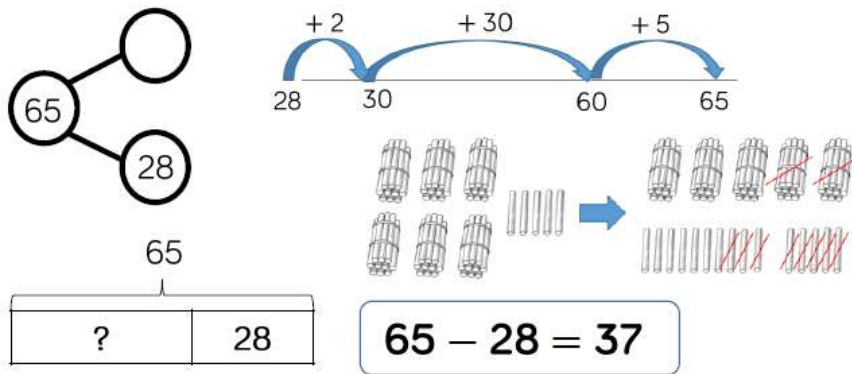
Subtraction

Year 1	Subtract 1-digit numbers within 10  $7 - 3 = 4$	Pupils experience subtraction as partitioning and reduction. Partitioning: part-whole models, bar models, ten frames and number shapes Reduction: Ten frames, number tracks, single bar models and bead strings Pupils are shown cubes and bar models with 2 bars to support finding the difference.
Year 1/2	Subtract 1 and 2-digit numbers to 20  $14 - 6 = 8$	Teachers highlight the importance of ten ones equalling one ten. Children find the number bond to 10 when partitioning the subtracted number. Children use number lines and tracks to support mental calculation.

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Year 2

Subtract 1 and 2-digit numbers to 100



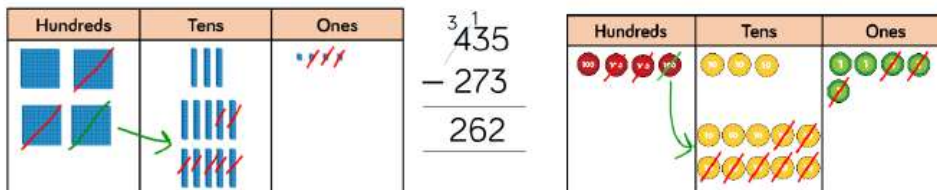
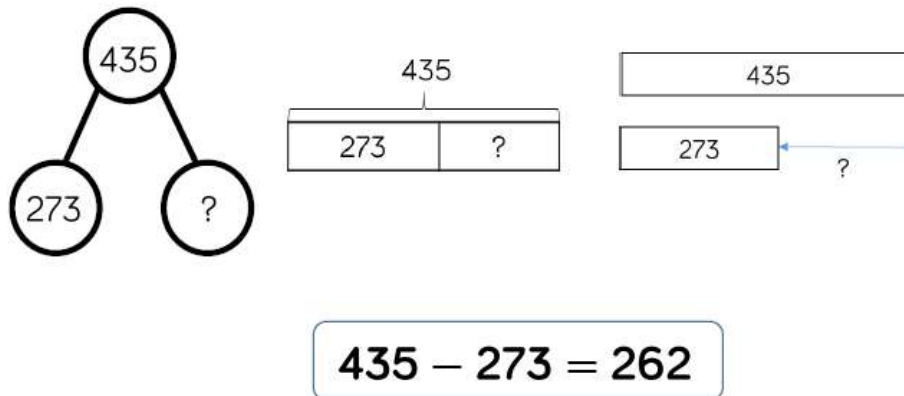
Children to use formal written addition alongside the use of concrete resources. As numbers become larger, straws become inefficient.

Children use a blank number line to count on and find the difference. As they become more efficient, they jump in multiples of 10.

In column subtraction, pupils consistently record exchanged digits on the top row of the calculation.

Year 3

Subtract numbers with up to 3 digits



Pupils write out their calculation alongside any concrete resources so they can see the links to the written column method.

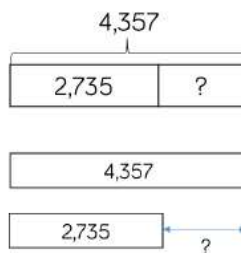
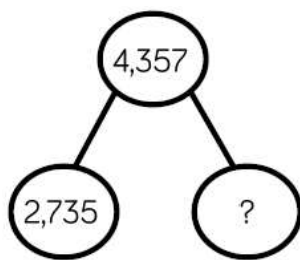
Children use plain counters on a place value grid to support learning.

In column subtraction, pupils consistently record exchanged digits on the top row of the calculation.

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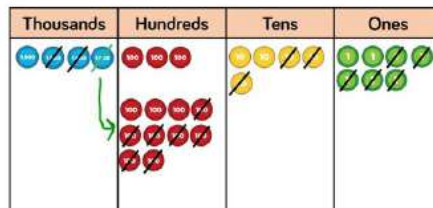
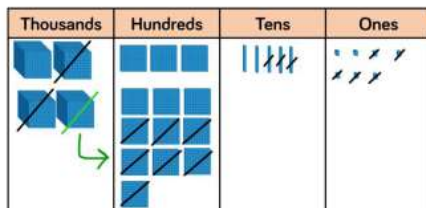
Year 4

Subtract numbers with up to 4 digits



$$\begin{array}{r} 4357 \\ - 2735 \\ \hline 1622 \end{array}$$

$$4,357 - 2,735 = 1,622$$



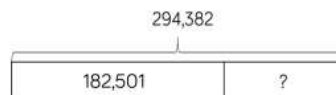
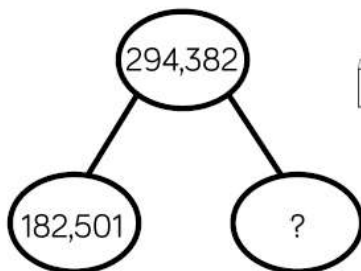
Children write out the calculation alongside any concrete resources they can see links to the written column method.

Plain counters on a place value grid are also used to support learning.

In column subtraction, pupils consistently record exchanged digits on the top row of the calculation.

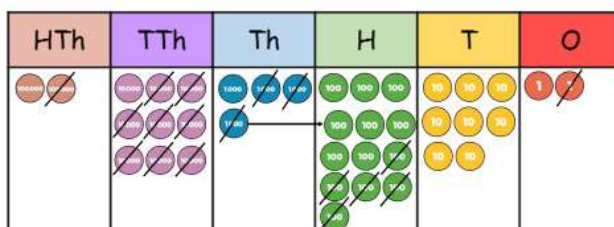
Year 5/6

Subtract numbers with more than 4 digits



$$\begin{array}{r} 294,382 \\ - 182,501 \\ \hline \end{array}$$

$$294,382 - 182,501 = 111,881$$

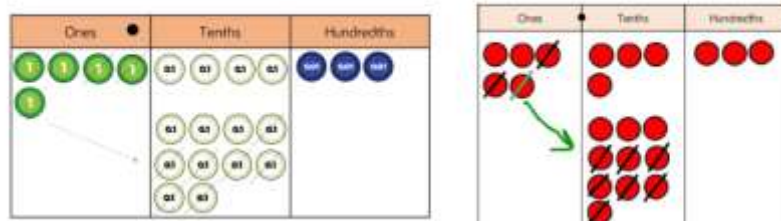
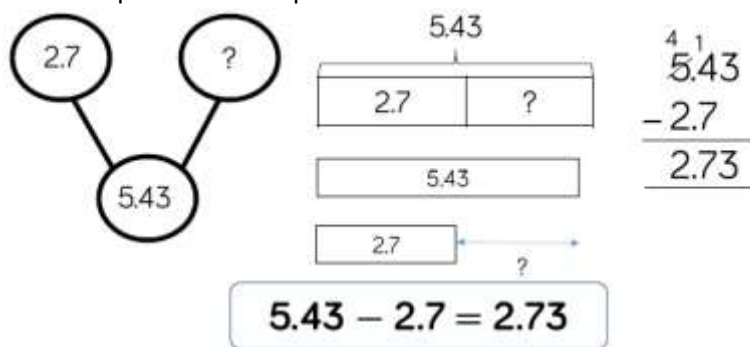


	2	9	3	1	8	2
-	1	8	2	5	0	1
	1	1	1	8	8	1

Children use the abstract, using the column method to subtract larger numbers.

In column subtraction, pupils consistently record exchanged digits on the top row of the calculation.

Subtract with up to 3 decimal places

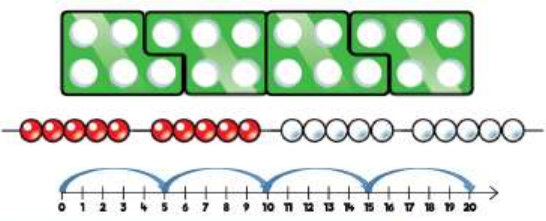
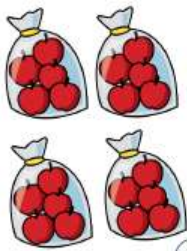
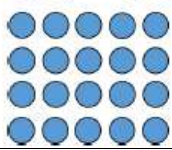
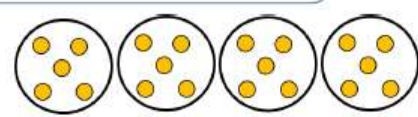
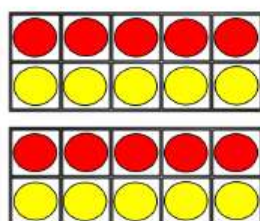
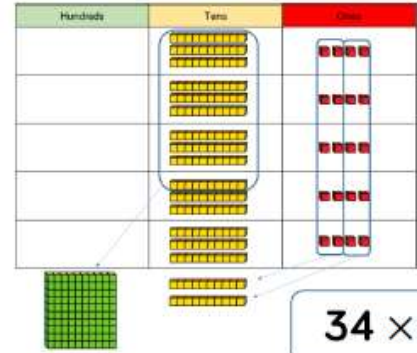


Pupils can represent subtraction calculations using place value counters and plain counters on a place value grid when subtracting decimals with 1,2 and 3 decimal places.

Children have experience of subtracting decimals with a variety of decimal places. This includes putting this into context when subtracting money and other measures.

In column subtraction, pupils consistently record exchanged digits on the top row of the calculation.

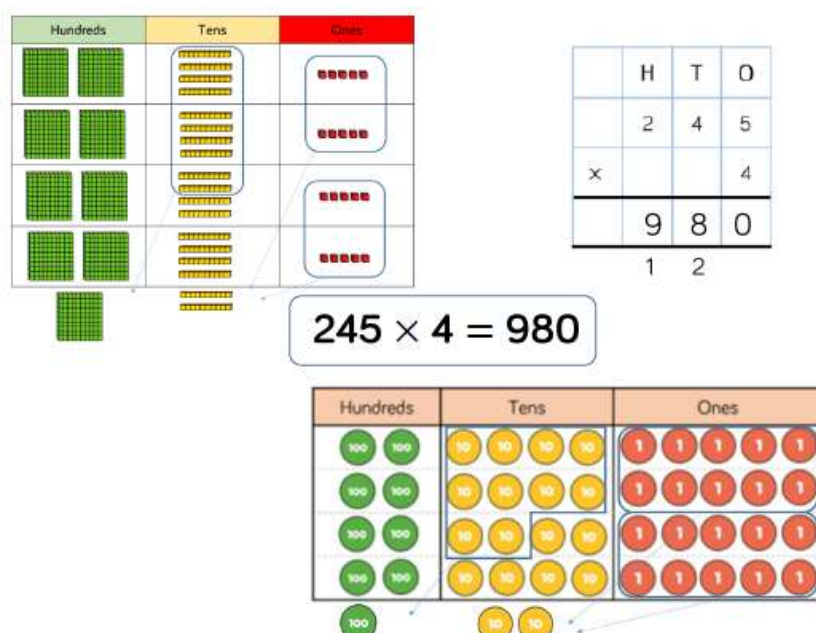
Multiplication

Year 1/2	Solve one-step problems using multiplication:  One bag holds 5 apples. How many apples do 4 bags hold? $5 + 5 + 5 + 5 = 20$$4 \times 5 = 20$$5 \times 4 = 20$	Children represent multiplication and repeated addition in many different ways. In Year 1, children use concrete and pictorial representations to solve problems and do not record multiplication formally. In Year 2, children are introduced to the multiplication symbol.																																												
Year 3	Multiply 2-digit numbers by 1-digit numbers <table border="1" data-bbox="737 896 1011 1160"><thead><tr><th></th><th>H</th><th>T</th><th>O</th></tr></thead><tbody><tr><td></td><td></td><td>3</td><td>4</td></tr><tr><td>x</td><td></td><td></td><td>5</td></tr><tr><td></td><td></td><td>2</td><td>0</td></tr><tr><td>+</td><td>1</td><td>5</td><td>0</td></tr><tr><td></td><td>1</td><td>7</td><td>0</td></tr></tbody></table>$34 \times 5 = 170$<table border="1" data-bbox="290 1276 579 1554"><thead><tr><th></th><th>H</th><th>T</th><th>O</th></tr></thead><tbody><tr><td></td><td></td><td>3</td><td>4</td></tr><tr><td>x</td><td></td><td></td><td>5</td></tr><tr><td></td><td></td><td>2</td><td>0</td></tr><tr><td>+</td><td>1</td><td>7</td><td>0</td></tr></tbody></table>		H	T	O			3	4	x			5			2	0	+	1	5	0		1	7	0		H	T	O			3	4	x			5			2	0	+	1	7	0	Place value counters are used to support understanding of the method rather than supporting the multiplication – pupils should use their times table knowledge. Pupils use Base 10 and place value counters to support understanding of the written method. Limit the number of exchanges at this point. Teachers support understanding of short multiplication by looking at the expanded method. In short multiplication, pupils consistently show regrouped digits below the equals sign.
	H	T	O																																											
		3	4																																											
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Year 4

Multiply 3-digit numbers by 1-digit numbers



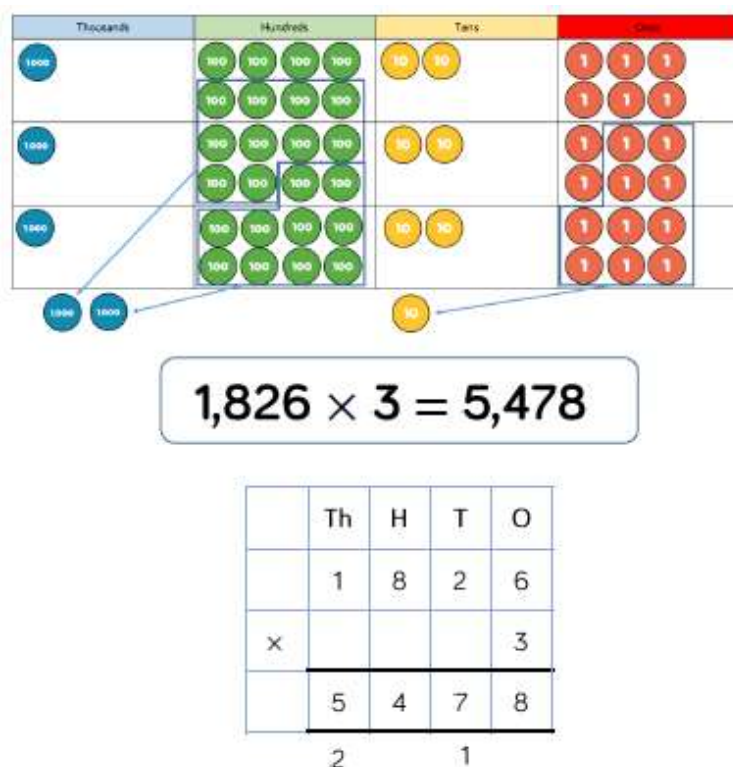
Pupils use Base 10 and place value counters to support understanding of the written method.

By the end of Year 4 pupils use short multiplication effectively.

In short multiplication, pupils consistently show regrouped digits below the equals sign.

Year 5

Multiply 4-digit numbers by 1 digit numbers



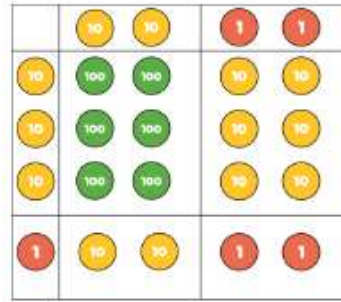
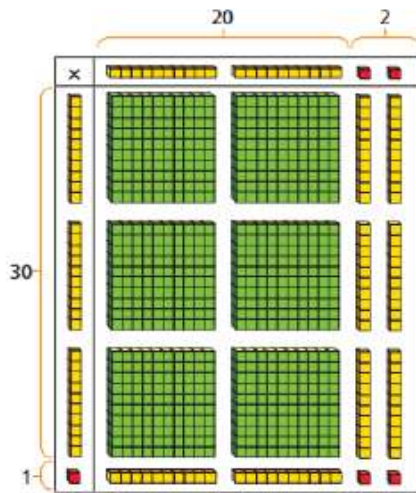
Pupils continue to use the formal written method of short multiplication. Place value counters and charts are used to support the understanding of what happens when regrouping.

Pupils struggling to use their times table knowledge use multiplication grids so that they can focus on the written method.

In short multiplication, pupils consistently show regrouped digits below the equals sign.

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Multiply 2-digit by 2-digit numbers



×	20	2
30	600	60
1	20	2

	H	T	O
×		2	2
		3	1
		2	2
	6	6	0
	6	8	2

$$22 \times 31 = 682$$

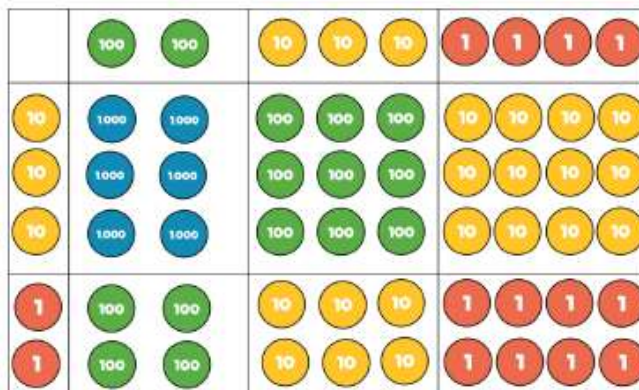
Pupils are exposed to and **initially** use the area model to help them to understand the size of numbers they are using.

Pupils use the grid method as an initial written method before moving on to the formal written multiplication method.

Pupils should efficiently use the formal written multiplication method by the end of Year 5.

In long multiplication, pupils consistently show regrouped digits below the equals sign.

Multiply 3-digit by 2-digit numbers



Th	H	T	O
	2	3	4
×		3	2
	4	6	8
17	10	2	0
7	4	8	8

$$234 \times 32 = 7,488$$

×	200	30	4
30	6,000	900	120
2	400	60	8

Pupils continue to use the area model to support their understanding of scale and size of numbers, Base 10 may be replaced with place value counters for efficiency.

Pupils to use the formal written method, seeing links with the grid method.

In long multiplication, pupils consistently show regrouped digits below the equals sign.

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Year 6

Multiply 4-digit by 2-digit numbers

TTh	Th	H	T	O
	2	7	3	9
×			2	8
2	1	9	1	2
5	4	7	8	0
7	6	6	9	2

2

5

3

7

1

1

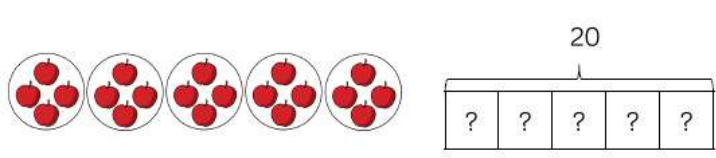
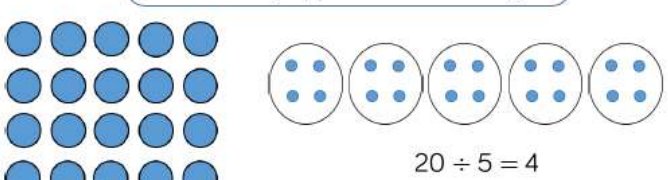
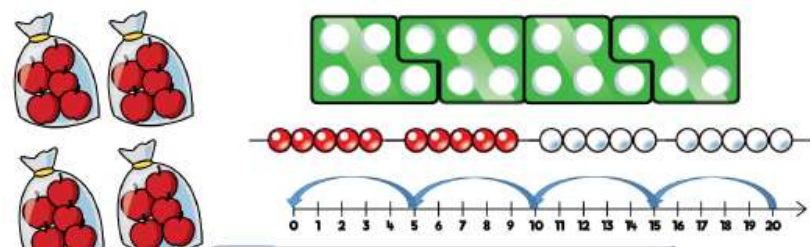
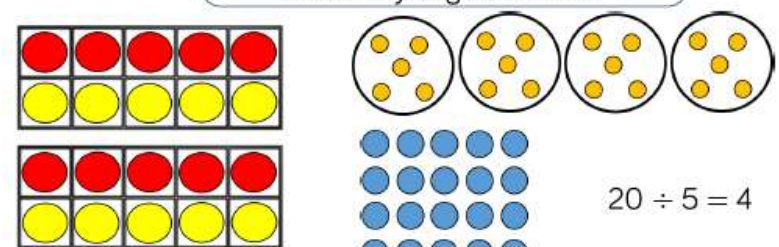
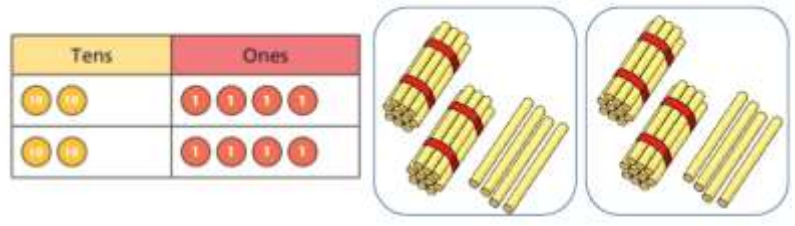
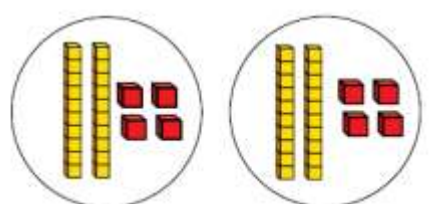
$$2,739 \times 28 = 76,692$$

Pupils are confident in using the formal written method of long multiplication.

Multiplication grids can still provide support for pupils struggling with times tables to ensure they can focus on the use of the method.

In long multiplication, pupils consistently show regrouped digits below the equals sign.

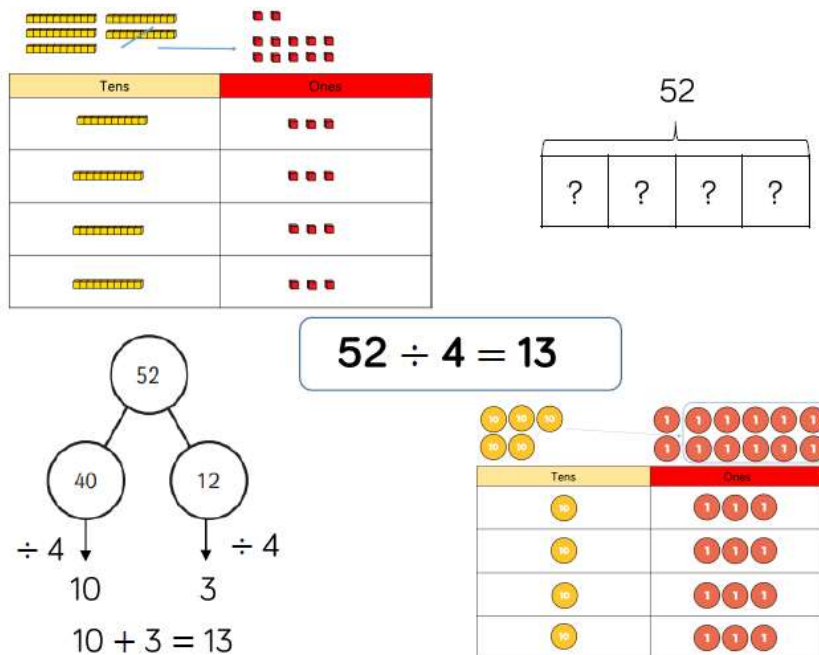
Division

Year 1/2	1-step problems using multiplication (sharing)  There are 20 apples altogether. They are shared equally between 5 bags. How many apples are in each bag?  $20 \div 5 = 4$	Children solve problems by sharing amounts into equal groups. In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record division formally. In Year 2, children are introduced to the division symbol.
Year 1/2	1-step problems using multiplication (grouping)  There are 20 apples altogether. They are put in bags of 5. How many bags are there?  $20 \div 5 = 4$	Children solve problems by grouping and counting the number of groups. Grouping encourages children to count in multiples and links to repeated subtraction on a number line. Pupils use concrete representations in fixed groups such a number shapes which helps to show the link between multiplication and division.
Year 2	Dividing 2-digits by 1-digit (no exchange)  $48 \div 2 = 24$ 	Children can use manipulatives that allow them to partition into tens and ones. Pupils use manipulatives including straws, Base 10 and place value counters to share numbers into equal groups. Use of part-whole models provides children with a clear written method that matches the concrete representation.

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Year 3

Dividing 2-digits by 1-digit (sharing with exchange)



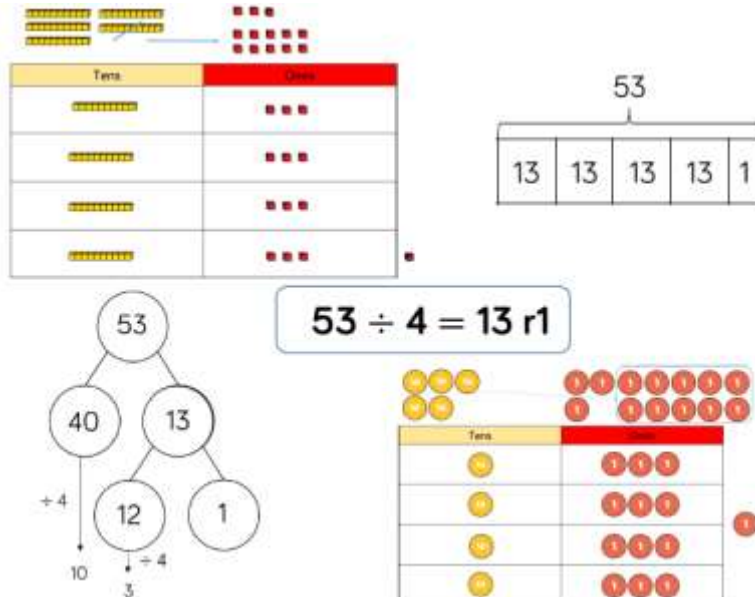
When dividing numbers involving an exchange, children can use Base 10 and place value counters to exchange one ten for ten ones.

Children should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows.

Flexible partitioning in a part-whole model supports this method,

Year 4

Dividing 2-digits by 1-digit (sharing with remainders)

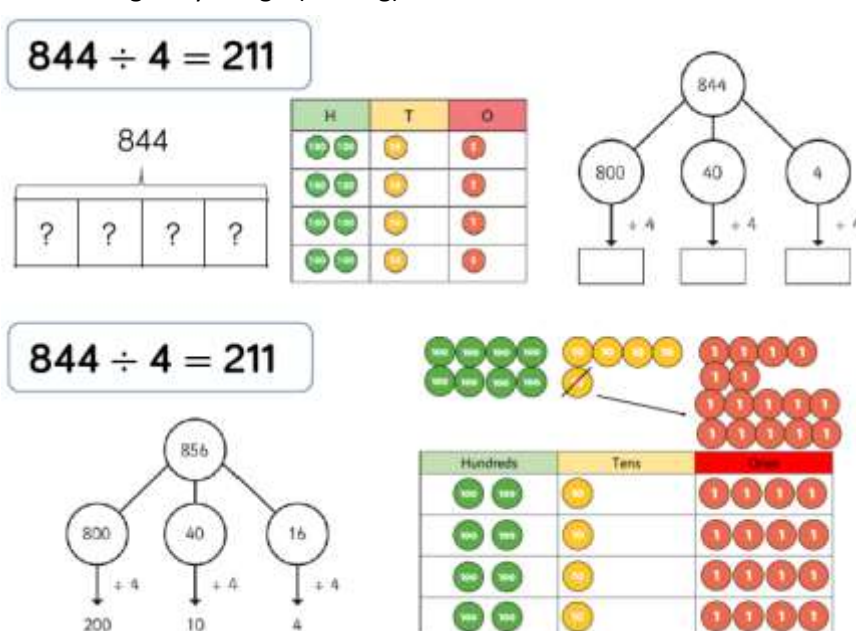


Children can use Base 10 and place value counters to exchange one ten for ten ones when dividing numbers with remainders

Pupils start with the equipment outside the place value grid to highlight remainders, as they will be left outside the grid once the equal groups have been made.

Year 4

Divide 3-digits by 1-digit (sharing)



Children continue to use place value counters to share 3-digit numbers into equal groups.

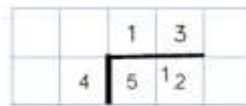
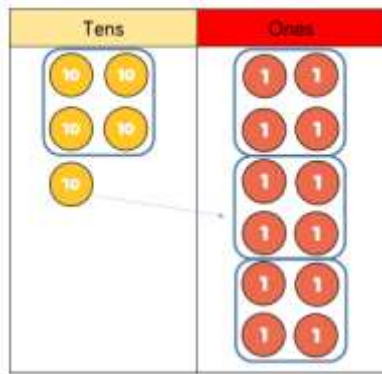
Children start with the equipment outside the place value grids before sharing the hundreds, tens and ones equally between the rows.

Pupils use flexible partitioning in a part-whole model to support this method.

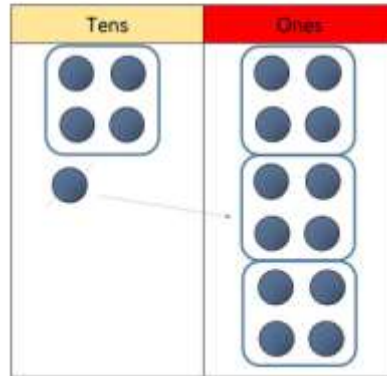
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Year 5

Divide 2-digits by 1-digit (regrouping)



$$52 \div 4 = 13$$



Pupils use the short division method using grouping. Starting with the largest place value, they group by the divisor.

Place value or blank counters are used to support pupils understanding of grouping. This highlights remainders clearly.

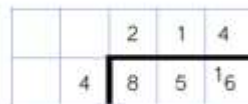
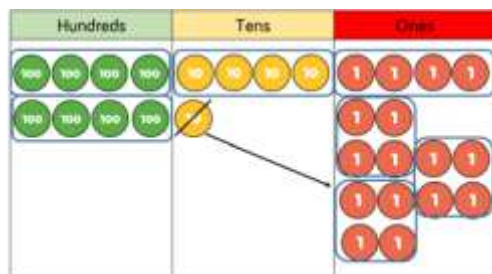
Children can initially draw their own counters and group them to support the pictorial method.

Teachers use precise language to prevent misconceptions: 'How many groups of 4 tens can we make?' and 'How many groups of 4 ones can we make?'

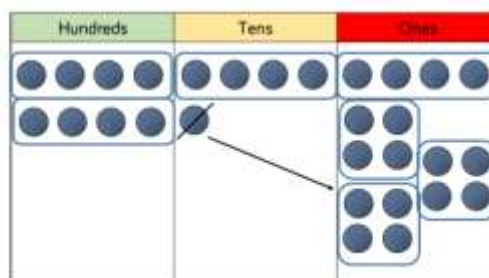
Pupils consistently exchange to the right and use superscript digits to show this.

Year 5

Divide 3-digits by 1 digit (grouping)



$$856 \div 4 = 214$$



Children continue to use grouping to support their understanding of short division when dividing a 3-digit number by a 1-digit number.

Place value counters or plain counters are used on a place value grid to support this understanding.

Pupils consistently exchange to the right and use superscript digits to show this.

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Year 6

Divide multi-digits by 2-digits

		0	3	6
	12	4	4	7
			3	2

$$432 \div 12 = 36$$

$$7,335 \div 15 = 489$$

	0	4	8	9
15	7	7	13	13
		3	3	5

15	30	45	60	75	90	105	120	135	150
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Children begin to divide up to 4-digits by 2-digits using short multiplication.

Children write out multiples to support their calculations with larger remainders.

Pupils can identify where remainders should be presented as decimals or fractions, or whether the quotient should be rounded.

Pupils consistently exchange to the right and use superscript digits to show this.

Year 6

Divide multi-digits by 2-digits (long division)

		0	3	6
1	2	4	3	2
	-	3	6	0
			7	2
	-		7	2
				0

$12 \times 1 = 12$
 $12 \times 2 = 24$
 $12 \times 3 = 36$
 $12 \times 4 = 48$
 $12 \times 5 = 60$
 $12 \times 6 = 72$
 $12 \times 7 = 84$
 $12 \times 8 = 96$
 $12 \times 9 = 108$
 $12 \times 10 = 120$

$$432 \div 12 = 36$$

$$7,335 \div 15 = 489$$

	0	4	8	9
15	7	3	3	5
-	6	0	0	0
	1	3	3	5
-	1	2	0	0
		1	3	5
-		1	3	5
				0

$1 \times 15 = 15$
 $2 \times 15 = 30$
 $3 \times 15 = 45$
 $4 \times 15 = 60$
 $5 \times 15 = 75$
 $10 \times 15 = 150$

Children divide by 2-digit numbers using long division.

Children write out multiples to support their calculations with larger remainders.

Children will also solve problems with remainders where the quotient can be rounded as appropriate.

Pupils consistently exchange to the right and use superscript digits to show this.

Year 6

Divide multi-digits by 2-digits (long division)

$$372 \div 15 = 24 \text{ r}12$$

		2	4	r	1	2
1	5	3	7	2		
	-	3	0	0		
			7	2		
	-		6	0		
			1	2		

$1 \times 15 = 15$
 $2 \times 15 = 30$
 $3 \times 15 = 45$
 $4 \times 15 = 60$
 $5 \times 15 = 75$
 $10 \times 15 = 150$

		2	4	$\frac{4}{5}$
1	5	3	7	2
	-	3	0	0
			7	2
	-		6	0
			1	2

$$372 \div 15 = 24 \frac{4}{5}$$

When a remainder is left at the end of a calculation, children can either leave it as a remainder or convert it to a fraction. They make decisions based on the context of the question.

Children answer questions where the quotient needs to be rounded according to the context.

Glossary of key terms for teachers:

Addend- a number to be added to another

Aggregation – combining two or more quantities or measures to find a total

Array – an ordered collection of counters, cubes or other item in rows and columns

Augmentation- increasingly a quantity of measure by another quantity

Commutative – numbers can be added or multiplied in any order

Complement – in addition, a number and its complement make a total. e.g. 300 is a complement to 700 to make 1000.

Difference – the numerical difference between two numbers is found by comparing the quantity in each group.

Difference – the numerical difference between two numbers is found by comparing the quantity in each group.

Dividend – in division, the number that is divided

Exchange – change a number or expression for another of equal value **in subtraction and division**

Factor – a number that multiplies with another to make a product

Minuend – a quantity or number from which another is subtracted

Multiplicand – in multiplication, a number to be multiplied by another

Partitioning – splitting a number into its component parts

Product – the result of multiplying one number by another

Quotient – the result of a division

Reduction – subtraction as take away

Regroup – change a number or expression for another of equal value **in addition and multiplication**

Remainder – the amount left over after a division when the divisor is not a factor of the dividend

Scaling – enlarging or reducing a number by a given amount, called the scale factor

Subitise – instantly recognise the number of objects in a small group without needing to count

Subtrahend – a number to be subtracted from another

Sum – the result of an addition

Total – the aggregate or the sum found by addition

Examples of formal written methods from the Primary Maths National Curriculum:

Addition and subtraction

789 + 642 becomes

$$\begin{array}{r} 789 \\ + 642 \\ \hline 1431 \\ \text{1 1} \end{array}$$

Answer: 1431

874 – 523 becomes

$$\begin{array}{r} 874 \\ - 523 \\ \hline 351 \end{array}$$

Answer: 351

932 – 457 becomes

$$\begin{array}{r} 8 \quad 12 \quad 1 \\ 932 \\ - 457 \\ \hline 475 \end{array}$$

Answer: 475

932 – 457 becomes

$$\begin{array}{r} 1 \quad 1 \\ 932 \\ - 457 \\ \hline 475 \\ \text{5 6} \end{array}$$

Answer: 475

Short multiplication

24 × 6 becomes

$$\begin{array}{r} 24 \\ \times 6 \\ \hline 144 \\ \text{2} \end{array}$$

Answer: 144

342 × 7 becomes

$$\begin{array}{r} 342 \\ \times 7 \\ \hline 2394 \\ \text{2 1} \end{array}$$

Answer: 2394

2741 × 6 becomes

$$\begin{array}{r} 2741 \\ \times 6 \\ \hline 16446 \\ \text{4 2} \end{array}$$

Answer: 16 446

Long multiplication

24 × 16 becomes

$$\begin{array}{r} 2 \\ 24 \\ \times 16 \\ \hline 240 \\ 144 \\ \hline 384 \end{array}$$

Answer: 384

124 × 26 becomes

$$\begin{array}{r} 1 \quad 2 \\ 124 \\ \times 26 \\ \hline 2480 \\ 744 \\ \hline 3224 \\ \text{1 1} \end{array}$$

Answer: 3224

124 × 26 becomes

$$\begin{array}{r} 1 \quad 2 \\ 124 \\ \times 26 \\ \hline 744 \\ 2480 \\ \hline 3224 \\ \text{1 1} \end{array}$$

Answer: 3224

Short division

98 ÷ 7 becomes

$$\begin{array}{r} 14 \\ 7 \overline{) 98} \\ \underline{7} \\ 28 \\ \underline{28} \\ 0 \end{array}$$

Answer: 14

432 ÷ 5 becomes

$$\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{) 432} \\ \underline{40} \\ 32 \\ \underline{30} \\ 2 \end{array}$$

Answer: 86 remainder 2

496 ÷ 11 becomes

$$\begin{array}{r} 45 \text{ r } 1 \\ 11 \overline{) 496} \\ \underline{44} \\ 56 \\ \underline{55} \\ 1 \end{array}$$

Answer: 45 $\frac{1}{11}$

Long division

432 ÷ 15 becomes

$$\begin{array}{r}
 28 \text{ r } 12 \\
 15 \overline{) 432} \\
 \underline{30 } \\
 132 \\
 \underline{120} \\
 12
 \end{array}$$

Answer: 28 remainder 12

432 ÷ 15 becomes

$$\begin{array}{r}
 28 \\
 15 \overline{) 432} \\
 \underline{30 } \quad 15 \times 20 \\
 132 \\
 \underline{120} \quad 15 \times 8 \\
 12
 \end{array}$$

$$\frac{12}{15} = \frac{4}{5}$$

Answer: $28 \frac{4}{5}$

432 ÷ 15 becomes

$$\begin{array}{r}
 28.8 \\
 15 \overline{) 432.0} \\
 \underline{30 } \quad \downarrow \\
 132 \\
 \underline{120} \quad \downarrow \\
 120 \\
 \underline{120} \quad \downarrow \\
 0
 \end{array}$$

Answer: 28.8